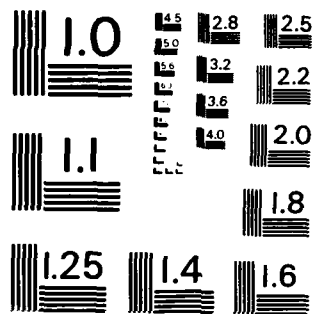


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DEPT OF MATERIALS SCIENCE AND ENG.. M OHTSU ET AL.  
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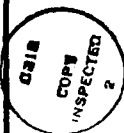
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# ANALYSIS OF PIEZOELECTRIC DISKS AND CYLINDERS USING FINITE ELEMENT METHOD

Masayasu Ohtsu\* and Kanji Ono

Department of Materials Science and Engineering,  
School of Engineering and Applied Science,  
University of California, Los Angeles, California  
90024.

\*Now at the Department of Civil Engineering,  
Faculty of Engineering, Kumamoto University,  
Kumamoto, JAPAN.

## ABSTRACT

Resonance characteristics of piezoelectric transducer elements were analyzed using the finite element method (FEM). We developed a computer program based on the axisymmetrical element formulation. Using this program, we determined resonance frequencies and vibration modes of cylindrical and disk-shaped elements of  $\text{BaTiO}_3$ , PZT-4 and PZT-5A ceramics. The resonance frequencies were obtained by the FEM analyses for the stress-free boundary condition. Resonance vibration modes were also determined and found to be quite complex. Dominant low frequency modes were "concave-extension" for a cylinder and "circular bending" and "radial-compression" for a disk. Most resonance frequencies were controlled primarily by the diameter of the element.

## INTRODUCTION

Frequency responses of acoustic emission (AE) transducers are one of major variables in AE studies. Piezoelectric transducers provide the best combination of low cost, high sensitivity, ease of handling and selective frequency responses. Although most of them are not suited for broad-band detection and their characteristics are not well characterized for use in basic studies of AE waveform analyses, they are the only choice for most AE experiments and applications.

A few solutions exist for three-dimensional piezoelectric bodies. Holland and EerNisse /1/ obtained solutions for thick piezoelectric disks using a variational method with trial functions based on Bessel functions. Bugdayci and Bogy /2/ discussed some particular solutions by means of the classical series expansion method. However, these results are neither explicit nor directly applicable to the analysis of AE transducers. In order to clarify frequency responses and vibration modes of AE transducers and to optimize the design of piezoelectric transducer elements, Ohtsu and Ono /3/ developed a numerical method, which is applicable for axisymmetric piezoelectric elements. They analyzed the resonance characteristics of piezoelectric AE transducers by means of the finite element method (FEM). Cylindrical, disk and conical elements of PZT-5A ceramic were investigated in their study.

Using the computer program, we evaluated the resonance characteristics of typical axisymmetrical piezoelectric elements for AE transducers. These are cylinders and disks made of  $\text{BaTiO}_3$ , PZT-4 and PZT-5A ceramics. The results are presented in parametric forms so that they can be used in designing AE sensors and compared with experimental results.

### FEM FORMULATION OF A PIEZOELECTRIC TRANSDUCER ELEMENT

In our previous work /3/, we developed a computer program for the analysis of vibrational characteristics of an axisymmetric piezoelectric element. It employed a linear triangular element for approximation of the displacement field. The first five resonance frequencies were obtained by this program.

We analyzed axisymmetrical piezoelectric elements, having cylindrical or disk-shaped geometry. The diameter-to-thickness ( $d/h$ ) ratios were varied between 0.25 and 6.0. Typical examples of finite element meshes of the axisymmetrical models analyzed are shown in Fig. 1. We used 16 to 32 triangular shaped elements and the stress-free boundary (free vibration) condition. The material constants of  $\text{BaTiO}_3$ , PZT-4 and PZT-5A ceramics used in this study were obtained from literature /4/.

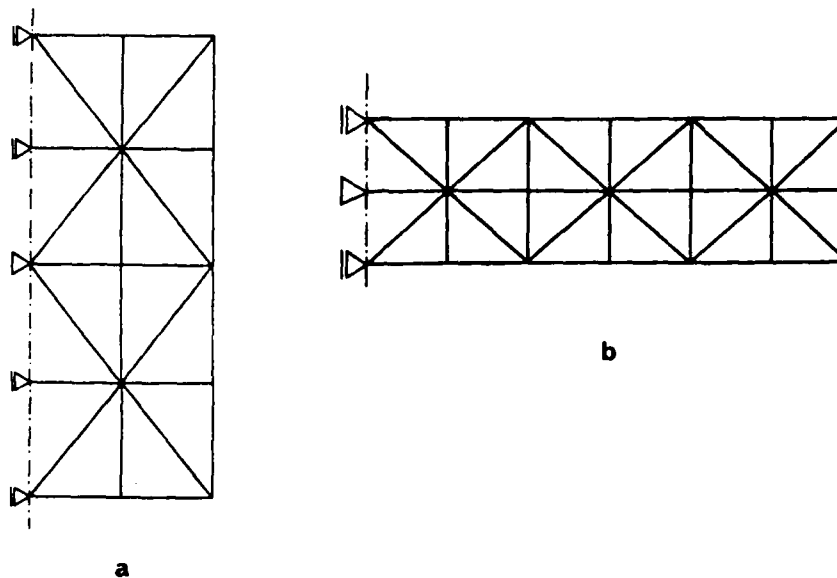


Fig. 1 Axisymmetrical models of transducer elements; (a) the cylindrical element (the number of nodes = 15, the number of elements = 16), b) the disk-shaped element (the number of nodes = 21, the number of elements = 24).

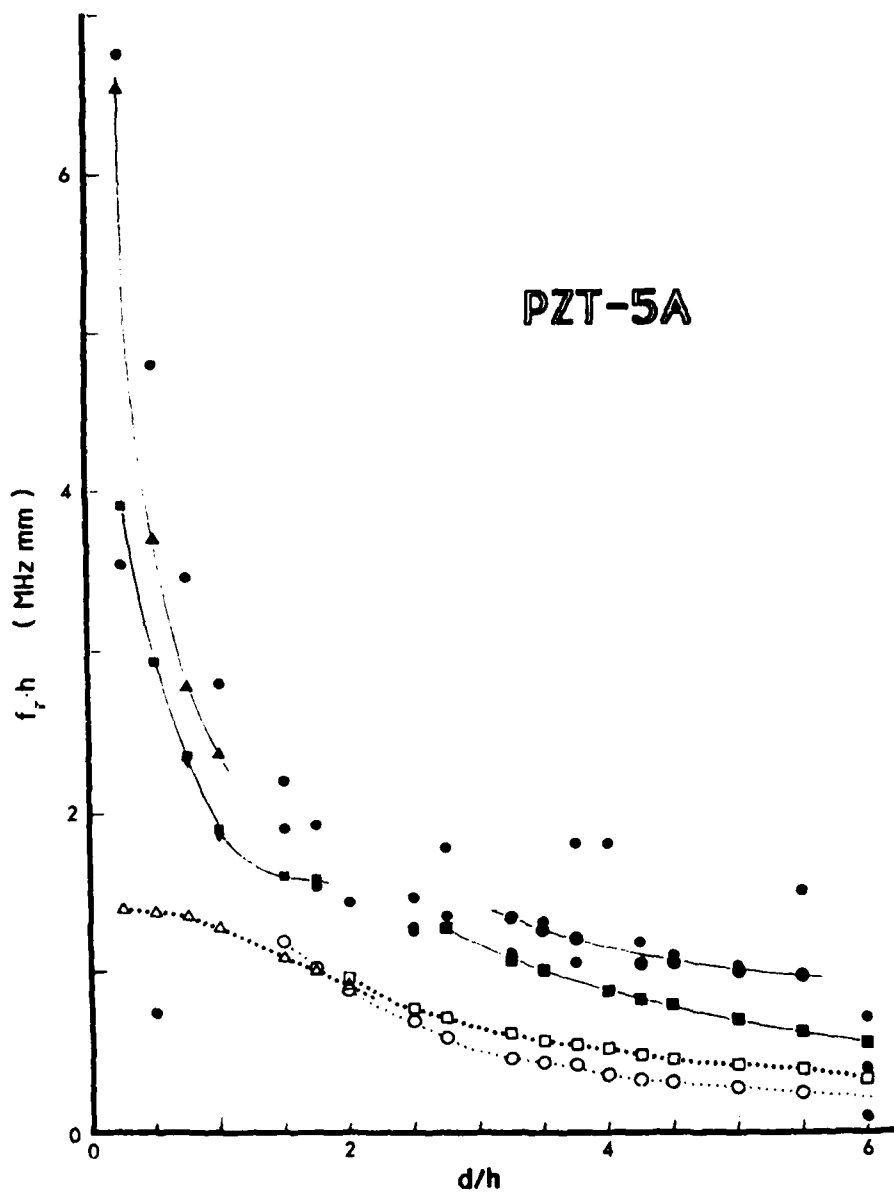


Fig. 2 Resonance frequencies ( $f_r$ ) times diameter ( $d$ ) versus diameter-to-thickness ratios ( $d/h$ ) for the case of freely vibrating PZT-5A cylinders or disks. Values for five modes are shown for each  $d/h$ . Those with identical vibration mode are connected with a curve.

## RESULTS

### 1. PZT-5A Ceramic

Resonance frequencies under the free vibration condition,  $f_r$ , were determined as a function of diameter-to-thickness ( $d/h$ ) ratios. In our previous study,  $d/h$  ratios of 0.78 and 6.57 were studied. Results are plotted in Fig. 2 in a normalized form as resonance frequency times thickness,  $f_r h$ , vs.  $d/h$ . Below  $d/h = 1.8$ , the first mode was observed at  $f_r h = 1$  to 1.4 MHzmm (shown by triangles in Fig. 2) and was of "concave-extension" type as shown in Fig. 3a. That is to say, the middle of a cylinder shrank radially and the thickness expanded axially, most expansion occurring in the center. Above  $d/h = 2.5$ , the first mode was of "circular bending" type, shown in Fig. 3b. For this mode,  $f_r h$  decreased gradually from 0.67 MHzmm at  $d/h = 2.5$  to 0.23 MHzmm at  $d/h = 5.5$ . These are shown by open circles in Fig. 2. The second mode in this  $d/h$  range was of "radial-compression" type, shown in Fig. 3c. The center of a disk is compressed and the diameter expanded uniformly. The resonance frequencies (marked by open squares in Fig. 2) were 20 to 70% higher than those of the radial-bending mode. Higher order modes in the "disk" region ( $d/h > 2.5$ ) acquire additional circular node(s) on disk faces. The radial expansion also becomes non-uniform, expanding more or less at the mid-thickness. The second mode at  $d/h = 0.75 - 1.0$  combines radius and thickness changes as shown in Fig. 3d. The third mode in this  $d/h$  range is of "convex-compression" type, similar to a barrel shape; that is, a larger mid-section diameter and reduced thickness at the center of the end faces. Higher order modes at lower  $d/h$  values contain more nodes on the side, and  $f_r h$  values become typically higher than 2 MHzmm.

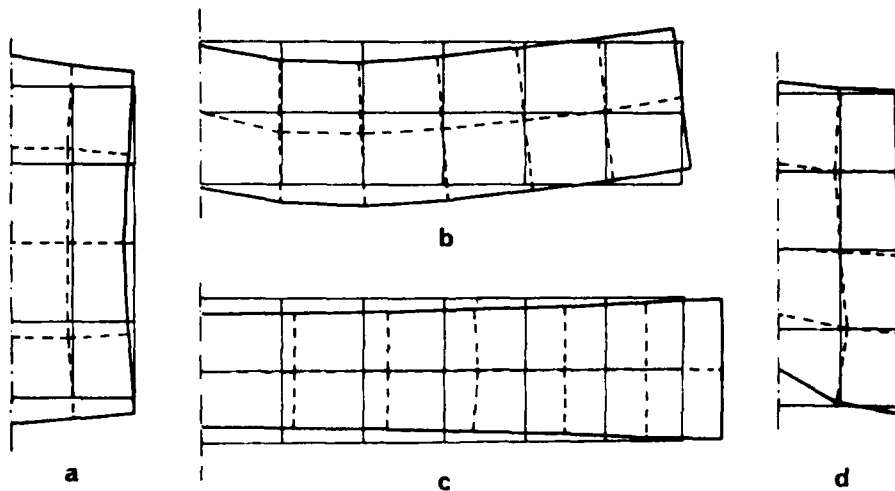


Fig. 3 Examples of resonance vibration modes of the cylindrical or disk-shaped element.



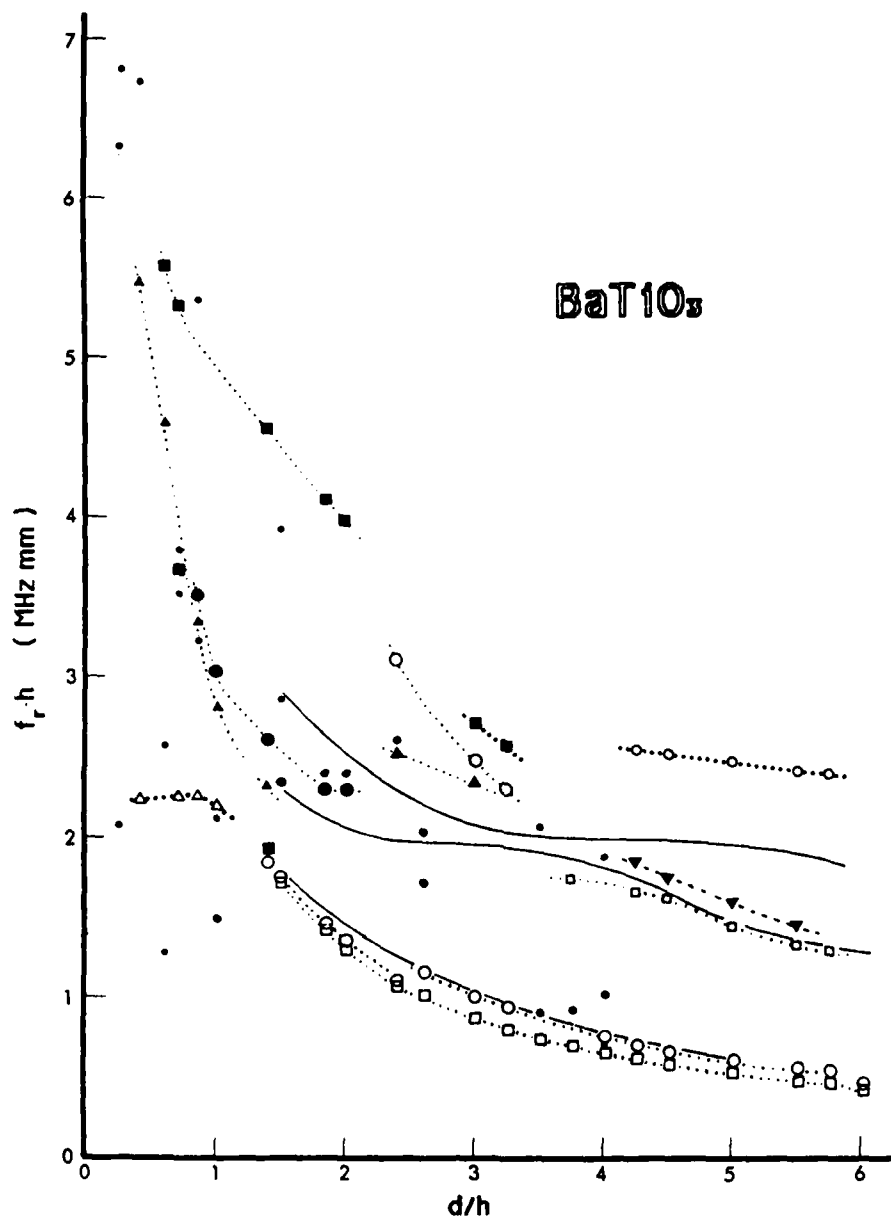


Fig. 4 Resonance frequencies ( $f_r$ ) times diameter ( $d$ ) versus diameter-to-thickness ratios ( $d/h$ ) for the case of freely vibrating BaTiO<sub>3</sub> cylinders or disks. Values for five modes are shown for each  $d/h$ . Those with identical vibration mode are connected with a curve. Dotted curves represent those calculated by the variational method.

## 2. BaTiO<sub>3</sub> Ceramic

A plot of  $f_r h$  vs.  $d/h$  is given in Fig. 4. General trends of resonance frequencies are similar to PZT-5A. However, the values of  $f_r h$  are higher than the corresponding quantities in PZT-5A. For example, the first resonance frequencies for BaTiO<sub>3</sub> at  $d/h = 2.5$  to  $5$  are about twice those in PZT-5A.

Three basic modes of vibration observed in PZT-5A were also found to exist in BaTiO<sub>3</sub>. The concave-extension mode was observed as the first mode at  $d/h = 0.4$  to  $0.85$  and as the third mode at  $d/h = 1.0$  (shown as triangles in Fig. 4). In contrast to PZT-5A, the first resonance was the radial-compression mode with the lowest  $f_r h$  values for  $d/h > 2.5$ . The second mode in BaTiO<sub>3</sub> was of the circular bending type with a slightly higher  $f_r h$  value.

In Fig. 4, three dotted curves indicate the three lowest resonance frequencies calculated by a variational method using Bessel functions /1/ as trial functions (1.5 to 5), the first curve is close to the second mode via the FEM method. Over the range of  $d/h$  calculated, the second curve is between the third and fourth modes at  $d/h > 3.5$ . Only a few points predicted by the present FEM method are close to the third curve obtained by the variational method.

## 3. PZT-4 Ceramic

The resonance frequencies for this material are given in Fig. 5. The results are basically identical to those of PZT-5A ceramic. However, individual vibration mode was not identified for this material.

## DISCUSSION

The present FEM analysis clearly demonstrates that three vibration modes dominate in the low frequency region. These are: concave-extension mode for cylinders; circular bending and radial-compression modes for disks. These shapes cannot be easily approximated by a one-dimensional parameter so that it is hardly surprising that one-dimensional theory /5/ cannot predict resonance frequencies in the range of  $d/h$  considered here. For thick disks of BaTiO<sub>3</sub>, calculations based on a variational method provided some resonance frequencies that agreed with the present analysis. Since the vibration modes were not given in the Holland-EerNisse calculation, the validity of the observed agreement cannot be ascertained. It should be noted that an appropriate trial function must be employed in this method. Those used in the calculations separated radial and axial components and cannot treat complex cross-coupling effects observed in the FEM analysis. This is a basic weakness of the variational method; more complicated geometries can easily be handled by FEM analysis. Thus, this is the most appropriate method for the study of piezoelectric elements.

When the data points with an identical vibration mode in Figs. 2 and 4 are connected, we find that  $f_r h$  generally decreases with  $d/h$ . This means that  $f_r$  is inversely proportional to  $d$  (when  $h$  is held constant). In one-dimensional theory, the radial resonance behaves in this manner. The present results suggest that most of the resonances are indeed controlled by the diameter of a piezoelectric element within the range of  $d/h$  employed (0.25 to 6.0). There are a few exceptions to this general trend, the most notable one being that for the first mode at  $d/h$  below unity (i.e., the concave-extension mode).

We have considered the case of free vibration. When a piezoelectric element is used as an AE sensor by coupling it to a test piece via viscous fluid, the present analysis provides a good approximation. On the other hand, bonding the element solidly to a protective facing or a test piece alters the boundary condition so that further analysis is required.

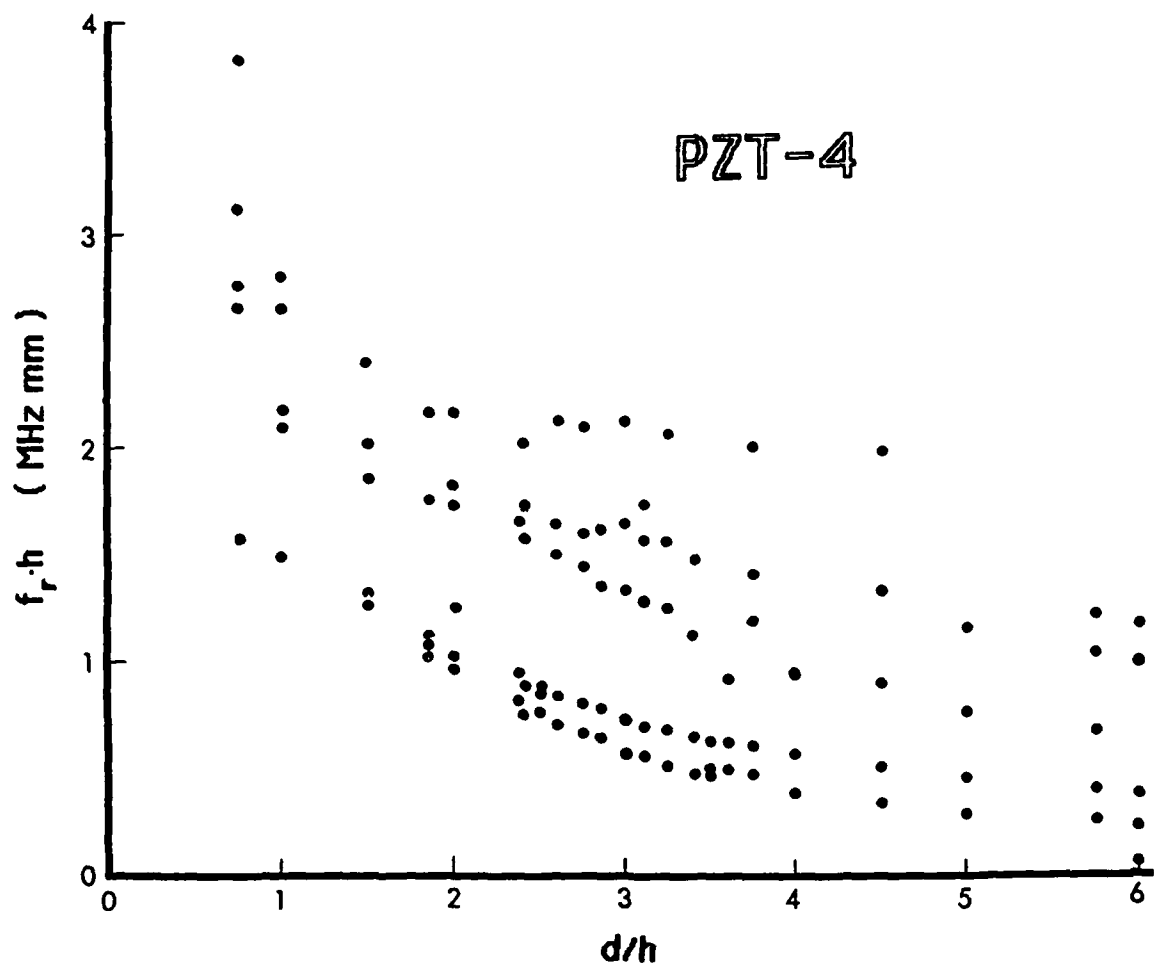


Fig. 5 Resonance frequencies ( $f_r$ ) times diameter ( $d$ ) versus diameter-to-thickness ratios ( $d/h$ ) for the case of freely vibrating PZT-4 cylinders or disks. Values for five modes are shown for each  $d/h$ .

### CONCLUSIONS

1. Using FEM analysis with axisymmetric elements, the resonance frequencies and vibration modes of piezoelectric disks and cylinders were determined for PZT-4, PZT-5A and BaTiO<sub>3</sub> ceramics.
2. Primary resonance of a cylinder is the concave-extension mode, while those of a disk are the circular bending and radial-compression modes. The former was the lowest resonance of PZT-5A disks and the latter was the lowest for BaTiO<sub>3</sub> disks.
3. The value of  $f_r h$  generally decreased with  $d/h$  for a single vibration mode. This implies that  $f_r$  is inversely proportional to  $d$  and that the diameter of a piezoelectric element is the primary controlling parameter for the resonance modes studied here.
4. One-dimensional theory cannot be used to predict resonance frequencies for the range of  $d/h$  between 0.25 and 6. The variational method predicts some resonances that are close to the present analysis. However, it is limited fundamentally by the need of providing an appropriate trial function. Thus, FEM is most suited for the analysis of piezoelectric elements.

### ACKNOWLEDGEMENTS

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